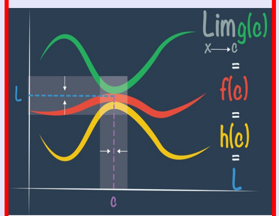
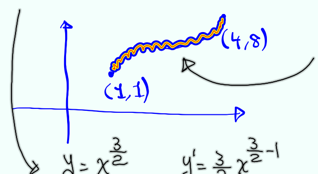
Calculus I

Lecture 47



Feb 19-8:47 AM

Find the length of the arc given by $y = x\sqrt{x}$ from $x=1$ to $x=4$.



$$L=\int_1^4 \sqrt{1+[f'(x)]^2}\,dx$$

$y = x^{\frac32}$

$y' = \frac32 x^{\frac32-1}$

$y' = \frac32 x^{1/2}$

$y' = \frac32\sqrt{x}$

$1+(y')^2 = 1+\left(\frac{3\sqrt{x}}{2}\right)^2$

$= 1+\frac94 x$

$$=\int_1^4\sqrt{1+\frac94 x}\,dx$$

$u = 1+\frac94 x$ $x=1,\ u=\frac{13}{4}$; $x=4,\ u=10$

$du = \frac94\,dx$

$\frac49\,du = dx$

$$\int_{\frac{13}{4}}^{10}\sqrt{u}\cdot\frac49\,du$$

$u^{1/2}$

⋮

✓ $\frac{1}{27}\left[80\sqrt{10}-13\sqrt{13}\right]$

$=\frac49\cdot\frac{u^{3/2}}{3/2}\Big|_{13/4}^{10}$

$=\frac{8}{27}u\sqrt{u}\Big|_{13/4}^{10} =$

$=\frac{8}{27}\left[\left(10\sqrt{10}-\frac{13}{4}\sqrt{\frac{13}{4}}\right)\right] = \frac{8}{27}\left[10\sqrt{10}-\frac{13}{8}\sqrt{13}\right]$

$=\frac{1}{27}\left[8\cdot10\sqrt{10}-\cancel{8}\cdot\frac{13}{\cancel{8}}\sqrt{13}\right] = \boxed{\frac{1}{27}\left[80\sqrt{10}-13\sqrt{13}\right]}$

Nov 21-8:29 AM

Evaluate $\int_1^3 (x^2-2x)\,dx$ using limit approach.

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\cdot\Delta x$$

$\Delta x = \frac{b-a}{n}$

$x_i = a + i\Delta x$

$a=1,\ b=3,\ f(x) = x^2-2x$

$\Delta x = \frac{3-1}{n} = \frac{2}{n}$

$x_i = 1 + i\cdot\frac{2}{n}$

$x_i = 1 + \frac{2i}{n}$

$$f(x_i) = \left(1+\frac{2i}{n}\right)^2 - 2\left(1+\frac{2i}{n}\right)$$

$$= \boxed{1} + \cancel{\frac{4i}{n}} + \frac{4i^2}{n^2} \boxed{-2} - \cancel{\frac{4i}{n}}$$

$$= \frac{4i^2}{n^2} - 1$$

$$\sum_{i=1}^{n} f(x_i)\cdot\Delta x = \sum_{i=1}^{n}\left(\frac{4i^2}{n^2}-1\right)\cdot\frac{2}{n} = \sum_{i=1}^{n}\left(\frac{8i^2}{n^3}-\frac{2}{n}\right)$$

$$= \sum_{i=1}^{n}\frac{8i^2}{n^3} - \sum_{i=1}^{n}\frac{2}{n}$$

$$= \frac{8}{n^3}\boxed{\sum_{i=1}^{n} i^2} - \frac{2}{n}\boxed{\sum_{i=1}^{n} 1}$$

$$= \frac{8}{n^3}\cdot\frac{n(n+1)(2n+1)}{6} - \frac{2}{n}\cdot n\cdot 1$$

$$= \frac{16n^3+\cdots}{6n^3} - 2$$

$$\int_1^3 (x^2-2x)\,dx = \lim_{n\to\infty}\left(\frac{16n^3+\cdots}{6n^3} - 2\right)$$

$$= \frac{16}{6} - 2 = \frac{8}{3} - 2 = \frac{8}{3} - \frac{6}{3} = \boxed{\frac{2}{3}}$$

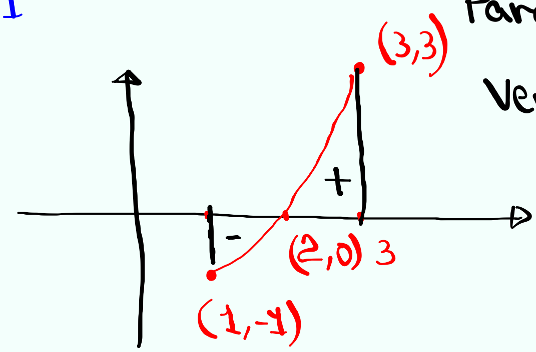
Nov 25-7:32 AM

$$\int_1^3 (x^2-2x)\,dx$$

$f(x) = x^2 - 2x$

Parabola, opens upward

Vertex at $(1,-1)$



$$\int_1^3 (x^2-2x)\,dx$$

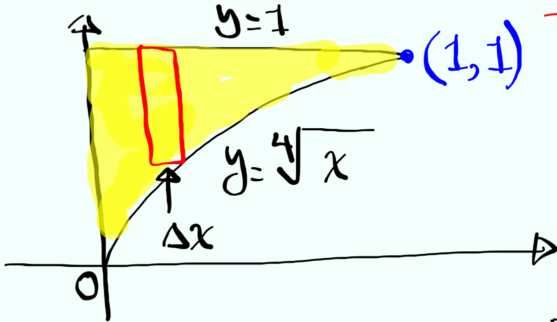
$$= \left(\frac{x^3}{3} - x^2\right)\Big|_1^3$$

$\frac{27}{3} - 9$ $\qquad$ $\frac{1}{3} - 1$

$$= \left(\frac{3^3}{3} - 3^2\right) - \left(\frac{1^3}{3} - 1^2\right) = 0 - \left(\frac{-2}{3}\right) = \boxed{\frac{2}{3}}$$

Nov 25-7:44 AM

Consider the shaded area below. (Not Scaled)



Top $y=1$

Bottom $y=\sqrt[4]{x}=x^{1/4}$

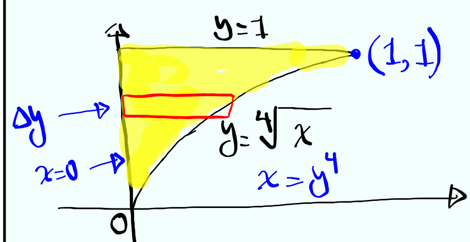
$$A=\int_0^1(1-x^{1/4})dx=\left(x-\frac{x^{5/4}}{5/4}\right)\Big|_0^1=\left(x-\frac{4}{5}x\sqrt[4]{x}\right)\Big|_0^1$$

$$=\left(1-\frac{4}{5}\right)-(0)$$

$$=\boxed{\frac{1}{5}}$$

Nov 25-7:49 AM

Consider the shaded area below. (Not Scaled)



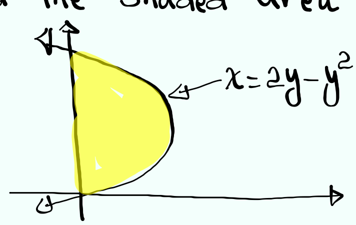
Right $x=y^4$

Left $x=0$

$$A=\int_0^1\Big[\underbrace{y^4-0}_{\text{Right}-\text{left}}\Big]dy=\frac{y^5}{5}\Big|_0^1=\frac{1^5}{5}-0=\boxed{\frac{1}{5}}$$

Do this on your own

Find the shaded area below



Nov 25-7:49 AM

Evaluate $\int_0^1 x^2\sqrt{x^3+1}\,dx$

Let $u = x^3+1$

$du = 3x^2\,dx$

$\frac{du}{3} = x^2\,dx$

$x=0 \rightarrow u = 0^3+1$

$u = 1$

$x=1 \rightarrow u = 1^3+1$

$u = 2$

$$= \int_1^2 \sqrt{u}\,\frac{du}{3}$$

$\sqrt{u} \rightarrow u^{1/2}$

$$= \frac{1}{3}\cdot\frac{u^{3/2}}{3/2}\Big|_1^2$$

$$= \frac{2}{9}\,u\sqrt{u}\Big|_1^2 = \frac{2}{9}\left[2\sqrt{2} - 1\sqrt{1}\right] = \boxed{\frac{4\sqrt{2}-2}{9}}$$

Nov 25-8:01 AM

Evaluate $\int_{-1}^{2} \overbrace{x^2\sqrt{x+2}}^{f(x)}\,dx$

x^2: Defined $(-\infty,\infty)$

$\sqrt{x+2}$: $x+2 \geq 0$

$x \geq -2$

$[-2,\infty)$

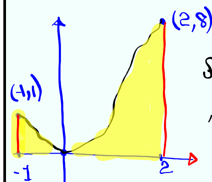
$f(-1) = (-1)^2\sqrt{-1+2} = 1\sqrt{1} = 1$

$f(2) = 2^2\sqrt{2+2} = 4\sqrt{4} = 8$

$x^2 \geq 0$

$\sqrt{x+2} \geq 0 \Rightarrow f(x) \geq 0$

above x-axis



Find the area below $f(x) = x^2\sqrt{x+2}$, above x-axis from $x=-1$ to $x=2$

$\int_{-1}^{2} x^2\sqrt{x+2}\,dx$

$u = x+2 \rightarrow u-2 = x$

$du = dx$

$x=-1 \rightarrow u=1$

$x=2 \rightarrow u=4$

$$\int_1^4 (u-2)^2\sqrt{u}\,du$$

$$= \int_1^4 [u^2-4u+4]\sqrt{u}\,du$$

$$= \int_1^4 \left(u^2\cdot u^{1/2} - 4u\cdot u^{1/2} + 4u^{1/2}\right)du$$

$$= \int_1^4 \left(u^{5/2} - 4u^{3/2} + 4u^{1/2}\right)du$$

Make sure to finish this

Nov 25-8:08 AM

Evaluate $\displaystyle\int_{-1}^{2} x^2 \boxed{\sqrt{x+2}}\,\boxed{dx}$

$u = \sqrt{x+2}$

$u^2 = x + 2 \rightarrow u^2 - 2 = x$

$2u\,du = dx$

$x = -1 \rightarrow u = 1$

$x = 2 \rightarrow u = 2$

$$\int_1^2 (u^2-2)^2 u \cdot 2u\,du$$

$$= 2\int_1^2 (u^4 - 4u^2 + 4)\cdot u^2\,du$$

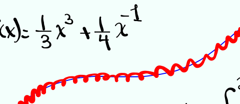
$$= 2\int_1^2 \left[u^6 - 4u^4 + 4u^2\right]du$$

Finish this and compare to last answer.

Nov 25-8:08 AM

Find the arc length of the curve given by $f(x) = \dfrac{x^3}{3} + \dfrac{1}{4x^1}$ on $1 \le x \le 2$.

$f(x) = \frac{1}{3}x^3 + \frac{1}{4}x^{-1}$



(2, ?)

(1, ?)

$$L = \int_1^2 \sqrt{1 + [f'(x)]^2}\,dx$$

$f'(x) = \frac{1}{3}\cdot 3x^2 - \frac{1}{4}x^{-2} = x^2 - \frac{1}{4}x^{-2}$

$1 + [f'(x)]^2 = 1 + \left(x^2 - \frac{1}{4}x^{-2}\right)^2$

$= 1 + x^4 - 2\cdot\cancel{x^2}\cdot\frac{1}{4}\cancel{x^{-2}} + \frac{1}{16}x^{-4}$

$= 1 + x^4 - \frac{1}{2} + \frac{1}{16}x^{-4}$

$= x^4 + \frac{1}{2} + \frac{1}{16}x^{-4} = \left(x^2 + \frac{1}{4}x^{-2}\right)^2$

$\sqrt{1 + [f'(x)]^2} = \sqrt{\left(x^2 + \frac{1}{4}x^{-2}\right)^2} = x^2 + \frac{1}{4}x^{-2}$

$$\int_1^2 \left(x^2 + \frac{1}{4}x^{-2}\right)dx$$

Make sure to finish it.

Nov 25-8:27 AM